NEARLY PERFECT TRANSMULTIPLEXERS USING COSINE-MODULATED FILTER BANKS

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ABSTRACT

In this paper, a new family of nearly perfect transmultiplexers based on cosine-modulated filter banks is presented. These filter banks are obtained using a new prototype filter design technique that controls the position of the passband cutoff frequency to minimize an objective function. The resulting transmultiplexers have good frequency selectivity, and they show low levels of intercarrier interference (ICI) and intersymbol interference (ISI). Several examples of such systems are given.

1. INTRODUCTION

Several popular communication applications can be described in terms of synthesis/analysis configuration (transmultiplexer) of subband transforms (Figure 1). Code division multiple access (CDMA), frequency division multiple access (FDMA), and time division multiple access (TDMA) communication schemes can be viewed from this perspective.

In particular, Orthogonal Frequency Division Multiplexing (OFDM) or Discrete Multitone transmission (DMT) modulation-based systems, propose transmultiplexers based on DFT filter banks [1]. However, it is well known that the selectivity of these filter banks is rather limited. The subchannel responses consist of a main lobe partly overlapping with immediate adjacent channels and high side lobes that extend over a wide frequency band. The level of the first side lobe is only –13 dB. Cosine-Modulated Filter Banks (CMFB) can replace traditional DFT banks because they provide better stopband attenuation and lower side lobes than in the DFT-based systems [2].

We propose a new prototype filter design technique to design nearly perfect transmultiplexers based on conventional cosine-modulated filter banks. The new systems have better adjacent channel isolation than DFT-

2. SCHEME OF COSINE MODULATION

We will consider the $M$-Channel transmultiplexer (figure 1) based on conventional cosine-modulation [4, 5] where all $M$ synthesis and analysis filters are obtained from a prototype filter. In this section, conventional cosine modulation by which the synthesis and analysis filters are derived is shown.

Let $p[n]$ be a symmetric linear-phase prototype filter with real coefficients designed as section 3 proposes. In conventional modulation, the real coefficients impulse response of synthesis $f_k[n]$ and analysis $h_k[n]$ filters (figure 2), $0 \leq n \leq N - 1$, are given by

$$f_k[n] = 2 \cdot p[n] \cdot \cos \left( \frac{(2k + 1) \pi}{2M} \left( n - \frac{N - 1}{2} \right) - \theta_k \right)$$

$$h_k[n] = 2 \cdot p[n] \cdot \cos \left( \frac{(2k + 1) \pi}{2M} \left( n - \frac{N - 1}{2} \right) + \theta_k \right)$$

$$0 \leq k \leq M - 1.$$
be the overall distortion transfer function given

\[ \phi = \left\| P(e^{j\omega /2M}) \right\| - 1/\sqrt{2} \]  

We must use a FIR filter design technique (by windowing, by means of optimum approximations,…) in order to guarantee that \( P^2(z) \) is approximately a 2Mth-band linear-phase FIR filter and the frequency response of the prototype filter satisfies approximately the power complementary property.

4. TRANSMULTIPLEXERS QUALITY EVALUATION.

Once the nearly perfect transmultiplexer is designed, the system's quality can be evaluated measuring the degree of closeness to perfect reconstruction. We use several quantities to measure two sources of error: crosstalk and magnitude distortion.

Crosstalk exists if there is signal leakage from one channel to another. This error can be quantified by means of the Intercarrier (interchannel) interference (ICI) as

\[ ICI = \max_{\omega \in \mathbb{R},r} \left( \sum_{l=0}^{M-1} \left| T_{r,l}(e^{j\omega}) \right|^2 \right) \]

where \( T_{r,l}(e^{j\omega}) \) is the crosstalk term, defined as

\[ T_{r,l}(e^{j\omega}) = \sum_{k=0}^{M-1} H_k(e^{j\omega} \cdot W_k^l) \cdot F_j(e^{j\omega} \cdot W_k^l) \]

The total crosstalk error for the \( j \)-th channel is defined as

\[ e_j = \int_{0}^{\pi} z^{j/M} \sum_{k=0}^{M-1} T_{r,k}(e^{j\omega})^2 d\omega \]

The maximum crosstalk error is

\[ e_{\max} = \max_{0 \leq j \leq M-1} e_j \]

Let \( \delta_T(z) \) be the overall distortion transfer function given by

\[ \delta_T(z) = \frac{1}{M} \cdot \sum_{k=0}^{M-1} F_k(z) \cdot H_k(z) \]

We measure the peak difference (maximum amplitude distortion) on this function by

\[ \delta_{pp} = \left( \left| T_0(e^{j\omega}) \right|_{\text{max}} - \left| T_0(e^{j\omega}) \right|_{\text{min}} \right)_{\omega \in [0,\pi]} \]

Even if both amplitude distortion and crosstalk error are zero, the \( j \)-th channel \( H_j(z) \cdot F_j(z) \) may still have errors in magnitude and phase compared with the ideal time delay. We can evaluate these errors by measuring the intersymbol interference (ISI), defined as.
\[ ISI = \max_r \left( \sum_n \left( t_{rr}[n] - \delta[n-n_d] \right)^2 \right), \]  

(12)

where \( t_{rr}[n] \) is the impulse response of the \( r \)-th channel, \( \delta[n] \) is the unit impulse, and \( n_d \) is a proper delay.

The powers of the interference components due to ICI and ISI are estimated by using the formulas proposed in [9].

We also introduce several quantitative measures in order to compare the performance of transmultiplexers designed. In this sense, we propose two ways of measuring the spread of the filter responses in time as well as in the frequency domain. The time-spread of a filter response \( f_i[n] \), \( 0 \leq n \leq N-1 \), characterises the time location property and is defined by [10]

\[
\sigma_{\tau_i}^2 = \frac{1}{E_i} \cdot \sum_n (n-\bar{n}_i)^2 \left| f_i[n] \right|^2
\]

(13)

The energy \( E_i \) and time center \( \bar{n}_i \) of the function \( f_i[n] \) are given as

\[
E_i = \sum_n \left| f_i[n] \right|^2 \quad \bar{n}_i = \frac{1}{E_i} \cdot \sum_n n \cdot \left| f_i[n] \right|^2
\]

(14)

The frequency-spread of a filter response \( f_i[n] \) represents the localisation measure given by

\[
\sigma_{\omega_i}^2 = \frac{1}{2 \cdot \pi \cdot E_i} \cdot \left| \int_{-\pi}^{\pi} (\omega-\bar{\omega}_i)^2 \cdot |F_i(\omega)|^2 d\omega \right|
\]

(15)

where

\[
F_i(\omega) = \sum_{n=0}^{N-1} f_i[n] \cdot e^{-j\omega n}
\]

(16)

\[
\bar{\omega}_i = \frac{1}{2 \cdot \pi \cdot E_i} \cdot \left| \int_{-\pi}^{\pi} \omega \cdot |F_i(\omega)|^2 d\omega \right|
\]

(17)

Figure 3. Magnitude response of the 129-length prototype filter (Hamming window).

Figure 4. \( F_k(e^{j\omega}) \) for the 8-channel transmultiplexer obtained with the prototype filter of figure 3.

Figure 5. \( F_0(e^{j\omega}) \) (periodic \( \pi/8 \)) for the 8-channel transmultiplexer obtained with the prototype filter of figure 3.
Figure 6. Magnitude response of the 129-length prototype filter (Blackman window).

Figure 7. $|F_k(e^{j\omega})|_{\omega}^{\{\text{periodic } \pi/16\}}$ for the 16-channel transmultiplexer obtained with the prototype filter of figure 6.

Figure 8. $|F_0(e^{j\omega})|_{\omega}^{\{\text{periodic } \pi/16\}}$ for the 16-channel transmultiplexer obtained with the prototype filter of figure 6.

Figure 9. Magnitude response of the 257-length prototype filter (Kaiser window).

Figure 10. $|F_k(e^{j\omega})|_{\omega}^{\{\text{periodic } \pi/32\}}$ for the 32-channel transmultiplexer obtained with the prototype filter of figure 9.

Figure 11. $|F_0(e^{j\omega})|_{\omega}^{\{\text{periodic } \pi/32\}}$ for the 32-channel transmultiplexer obtained with the prototype filter of figure 9.
5. CONCLUSIONS

In this paper, a new prototype filter design technique for nearly perfect transmultiplexers based on cosine-modulated filter banks have been presented. The problem of designing the prototype filter is formulated as a problem of optimizing the passband edge to minimize an objective function. The resulting transceivers have good frequency selectivity, and the results illustrate that small ICI and ISI can be obtained. Several examples are given to confirm the validity of the proposed technique.

6. REFERENCES


Table I. Transmultiplexers quality comparison.

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<th>ISI (dB)</th>
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